PROBLEMS—NOTES.

16. Skeleton Division. This note may be regarded as a continuation of note 2 (*1921*, 37). In *Nieuw Tijdschrift voor Wiskunde,* volume 8, 1920–21, Professor F. Schuh, of Delft, proposed (page 64) two arithmetical puzzles; one of them is herewith reproduced.

"The following calculation represents the development of an irreducible fraction to a repeating decimal:

```
                       _________
XXXXXXX) XXXXXXXX (XX.XXXXXXXXXX
         XXXXXX
         ------
         XXXXXXX
          XXXXXX
          -------
          XXXXXXX
          XXXXXXX
          -------
           XXXXXXX
            XXXXXX
            -------
            XXXXXXX
            XXXXXXX
            -------
             XXXXXXX
             XXXXXXX
             -------
              XXXXXXX
              XXXXXXX
              -------
               XXXXXXX
               XXXXXXX
               -------
                    XXXXXX
                    XXXXXX
                    ------
                    XXXXXX
```

Crosses indicate digits—all of which are possible. The repeating digits are shown by a line over the crosses. What irreducible fraction has been developed?"

ARC.

17. Curves of Pursuit (Compare *1921*, 54–61, 91–97, 184–185). I see that the AMERICAN MATHEMATICAL MONTHLY has been interesting itself in early examples of the Curve of Pursuit. Perhaps some of its readers may like these notes on four examples of it which appeared in England before 1750.

The general problem of the form of the curve, the pursued travelling in a straight line, is discussed in T. Simpson's *New Treatise of Fluxions,* London, 1737, p. 170, that is five years after the appearance of Bouguer's memoir.

A problem on it, by Robert Heath, appeared in *Ladies Diary,* 1742 and 1743 (Leybourn's reprint, vol. 1, p. 323), in the following form: "Admiral Vernon

sailing on a south course, from Jamaica to Carthagena, sees Don Blass right before him steering west, along the shore. He now continually bears directly upon him in a right line; when coming up with him, it appears that the Don had sailed 8 leagues during the chace, and that the said admiral was 7 leagues distant from him when the chace began: Now, supposing each ship's motion to be uniform during the whole chace, to find from thence the distance sail'd by Admiral Vernon?"

The following problem was propounded by Thomas Perryam in *Gentleman's Diary*, 1749: "A Captain of a Privateer seeing a Merchant Ship at S.S.E. sailing due West, continually bears directly upon her in a right Line; and coming up with her, it appear'd that the Merchant's Ship sail'd 30 Miles during the Chace, and that the said Privateer was 21 Miles distant from her when the Chace first began: Now supposing each Ship's Motion to be uniform during the whole Chace, To find from thence the Distance sail'd by the said Privateer, its East and West Departure, and also their Difference of Latitude when they were North and South of each other." A solution by fluxions was given in 1750.

Another question on the subject was propounded by John Ash in *Ladies Diary*, 1748, (Leybourn's reprint, vol. 2, p. 15) in the following form: "A spider, at one corner of a semi-circular pane of glass, gave uniform and direct chase to a fly, moving uniformly along the curve before him; the fly was 30° from the spider at the first setting out, and was taken by him at the opposite corner. What is the ratio of both their uniform motions[1]?" On this question the editor said "Mr. Landen sent us a true method [of solution]; but the calculus being so operose, it was not wrought out. And no method appearing to us yet elegant enough for a place, it will be next year before we shall have time to catch the solution." No further remarks on it appear in subsequent numbers of the *Diary*.

It would seem then that before 1750 such questions had become sufficiently familiar in England to appear in popular journals.

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18. Radius of the sphere circumscribing a tetrahedron. A member of the Association proposed the following problem which during the past century has been frequently solved: "Find, in terms of the lengths of the edges of a tetrahedron, the radius, R, of the circumscribing sphere." Denoting the pairs of opposite edges of the tetrahedron by a, a_1; b, b_1; c, c_1, L.N.M.Carnot derived[2] in 1806 the relation

$$\begin{aligned}4R^2(a_1^4a^2 + a^4a_1^2 + b_1^4b^2 + b^4b_1^2 + c_1^4c^2 + c^4c_1^2 + a^2b_1^2c_1^2 + c^2a_1^2b_1^2 \\ + b^2a_1^2c_1^2 + a^2b^2c^2 - a^2b^2b_1^2 - b^2c^2b_1^2 - b^2a_1^2b_1^2 - a^2a_1^2b_1^2 - b^2b_1^2c_1^2 \\ - c^2b_1^2c_1^2 - a^2b^2a_1^2 - a^2c^2a_1^2 - b^2c^2c_1^2 - a^2c^2c_1^2 - a^2a_1^2c_1^2 - c^2a_1^2c_1^2) \\ + 2b^2c^2b_1^2c_1^2 + 2a^2c^2a_1^2c_1^2 + 2a^2b^2a_1^2b_1^2 - a^4a_1^4 - b_1^4b^4 - c_1^4c^4 = 0.\end{aligned}$$

[1] Compare the notes on problem 2801 below.—Editor.

[2] *Mémoire sur la Relation qui existe entre les distances respectives de cinq points quelconques pris dans l'espace.* Paris, 1806, p. 11.